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Some properties of the regular asynchronous systems

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Abstract

The asynchronous systems are the models of the asynchronous circuits from the digital electrical engineering. An asynchronous system f is a multi-valued function that assigns to each admissible input $u : \mathbf{R} \rightarrow \{0,1\}^m$ a set $f(u)$ of possible states $x \in f(u), x : \mathbf{R} \rightarrow \{0,1\}^n$. A special case of asynchronous system consists in the existence of a Boolean function $\Upsilon : \{0,1\}^n \times \{0,1\}^m \rightarrow \{0,1\}^n$ such that $\forall u, \forall x \in f(u)$, a certain equation involving Υ is fulfilled. Then Υ is called the generator function of f (Moisil used the terminology of network function) and we say that f is generated by Υ. The systems that have a generator function are called regular.

Our purpose is to continue the study of the generation of the asynchronous systems that was started in [2], [3].

Keywords: asynchronous system, regularity, generator function

1 Preliminaries

Notation 1. *Let be the arbitrary set* M. *The following notation will be useful:* $P^*(M) = \{M' | M' \subset M, M' \neq \emptyset\}$.

Definition 2. *The set* $\mathbf{B} = \{0,1\}$, *endowed with the order* $0 \leq 1$ *and with the usual laws* $^-, \cdot, \cup, \oplus$, *is called the* ***binary Boole algebra****.*

Definition 3. *The* ***initial value*** $x(-\infty + 0) \in \mathbf{B}^n$ *of the function* $x : \mathbf{R} \rightarrow \mathbf{B}^n$ *is defined by* $\exists t' \in \mathbf{R}, \forall t < t', x(t) = x(-\infty + 0)$.

Definition 4. *The* ***characteristic function*** $\chi_A : \mathbf{R} \rightarrow \mathbf{B}$ *of the set* $A \subset \mathbf{R}$ *is given by* $\forall t \in \mathbf{R}, \chi_A(t) = \begin{cases} 1, t \in A \\ 0, \ else \end{cases}$.

Notation 5. *We use the notation* $Seq = \{(t_k) | t_k \in \mathbf{R}, k \in \mathbf{N}, \ t_0 < ... < t_k < ...$ *is unbounded from above*$\}$.

Definition 6. *A function* $x : \mathbf{R} \rightarrow \mathbf{B}^n$ *is called* $n-$***signal****, shortly* ***signal*** *if* $\mu \in B^n$ *and* $(t_k) \in Seq$ *exist such that*

$$x(t) = \mu \cdot \chi_{(-\infty,t_0)}(t) \oplus x(t_0) \cdot \chi_{[t_0,t_1)}(t) \oplus ... \oplus x(t_k) \cdot \chi_{[t_k,t_{k+1})}(t) \oplus ... \quad (1)$$

The set of the $n-$*signals is denoted by* $S^{(n)}$.

Remark 7. *Let be* $x : \mathbf{R} \rightarrow \mathbf{B}^n, u : \mathbf{R} \rightarrow \mathbf{B}^m$. *Instead of* $x \times u : \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{B}^n \times \mathbf{B}^m$ *we define the function* $x \times u$, *many times denoted by* (x,u), *as* $x \times u : \mathbf{R} \rightarrow \mathbf{B}^n \times \mathbf{B}^m$ *due to the existence of a unique time variable* $t \in \mathbf{R}$. *Between the consequences derived from here we have the identifications* $S^{(n)} \times S^{(m)} = S^{(n+m)}$ *and* $P^*(S^{(n)}) \times P^*(S^{(m)}) = P^*(S^{(n+m)})$.

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Definition 8. *The* ***left limit*** *$x(t-0)$ of $x(t)$ from (1) is the $\mathbf{R} \to \mathbf{B}^n$ function defined as $x(t-0) = \mu \cdot \chi_{(-\infty,t_0]}(t) \oplus x(t_0) \cdot \chi_{(t_0,t_1]}(t) \oplus ... \oplus x(t_k) \cdot \chi_{(t_k,t_{k+1}]}(t) \oplus ...$*

Definition 9. *Let be $U \in P^*(S^{(m)})$. A multi-valued function $f : U \to P^*(S^{(n)})$ is called* ***asynchronous system****, shortly* ***system****. Any $u \in U$ is called* ***(admissible) input*** *and the functions $x \in f(u)$ are called* ***(possible) states****.*

Remark 10. *The asynchronous systems are the models of the asynchronous circuits. The multi-valued character of the cause-effect association is due to the statistical fluctuations in the fabrication process, the variations in the ambiental temperature, the power supply etc. Sometimes the systems are given by equations and/or inequalities.*

Definition 11. *The* ***initial state function*** *of f is by definition the function $i_f : U \to P^*(\mathbf{B}^n), \forall u \in U, i_f(u) = \{x(-\infty+0)|x \in f(u)\}$.*

Definition 12. *The function $\rho : \mathbf{R} \to \mathbf{B}^n$ is called* ***progressive*** *if $(t_k) \in Seq$ exists such that $\rho(t) = \rho(t_0) \cdot \chi_{\{t_0\}}(t) \oplus ... \oplus \rho(t_k) \cdot \chi_{\{t_k\}}(t) \oplus ...$ and $\forall i \in \{1,...,n\}$, the set $\{k|k \in \mathbf{N}, \rho_i(t_k) = 1\}$ is infinite. The set of the progressive functions is denoted by P_n.*

Notation 13. *Let be $\Upsilon : \mathbf{B}^n \times \mathbf{B}^m \to \mathbf{B}^n, u \in S^{(m)}$, $\mu \in \mathbf{B}^n$ and $\rho \in P_n$. The solution of the equation*

$$\begin{cases} x(-\infty+0) = \mu \\ \forall i \in \{1,...,n\}, x_i(t) = \begin{cases} \Upsilon_i(x(t-0), u(t-0)), if\ \rho_i(t) = 1 \\ x_i(t-0), otherwise \end{cases} \end{cases} \quad (2)$$

is denoted by $\Upsilon^{-\rho}(t,\mu,u)$.

Definition 14. *The system $\Sigma^-_\Upsilon : S^{(m)} \to P^*(S^{(n)}), \forall u \in S^{(m)}, \Sigma^-_\Upsilon(u) = \{\Upsilon^{-\rho}(t,\mu,u)|\mu \in \mathbf{B}^n, \rho \in P_n\}$ is called the* ***universal regular asynchronous system*** *that is generated by the function Υ.*

Definition 15. *The system f is called* ***regular*** *if Υ exists such that $\forall u \in U, f(u) \subset \Sigma^-_\Upsilon(u)$. If so, Υ is called the* ***generator function*** *of f and we also say that Υ* ***generates*** *f.*

Remark 16. *Equation (2) shows how the circuits compute asynchronously the Boolean function Υ : the computation is made at the discrete time instances $\{t_k|k \in \mathbf{N}, \exists i \in \{1,...,n\}, \rho_i(t_k) = 1\}$ on these coordinates Υ_i for which $\rho_i(t_k) = 1$. The models of these circuits, the systems f with the generator function Υ, have the remarkable property that a function $\pi_f : W_f \to P^*(P_n)$ exists, $W_f = \{(x(-\infty+0),u)|u \in U, x \in f(u)\}$ such that $\forall u \in U, f(u) = \{\Upsilon^{-\rho}(t,\mu,u)|\mu \in i_f(u), \rho \in \pi_f(\mu,u)\}$. π_f is called the* ***computation function*** *of f. For f regular, Υ and π_f are not unique.*

2 Subsystems

Definition 17. *The system f is called a* ***subsystem*** *of $g : V \to P^*(S^{(n)})$, $V \in P^*(S^{(m)})$ and we write $f \subset g$, if $U \subset V$ and $\forall u \in U, f(u) \subset g(u)$.*

Remark 18. *We interpret $f \subset g$ in the following way: the systems f and g model the same circuit, but the model represented by f is more precise than the model represented by g.*

Theorem 19. *The function Υ and the regular systems $f \subset \Sigma^-_\Upsilon$, $g \subset \Sigma^-_\Upsilon$ are given. We denote by $i_g : V \to P^*(\mathbf{B}^n)$ the initial state function and by $\pi_g : W_g \to P^*(P_n)$ the computation function of g. The following statements are equivalent:*

a) $f \subset g$

b) $U \subset V$ and $\forall u \in U, i_f(u) \subset i_g(u)$ and $\forall u \in U, \forall \mu \in i_f(u), \forall \rho \in \pi_f(\mu,u), \exists \rho' \in \pi_g(\mu,u), \Upsilon^{-\rho}(t,\mu,u) = \Upsilon^{-\rho'}(t,\mu,u)$.

3 Dual systems

Definition 20. *The* ***dual*** *function* $\Upsilon^* : \mathbf{B}^n \times \mathbf{B}^m \to \mathbf{B}^n$ *of* Υ *is defined by* $\forall(\mu,\nu) \in \mathbf{B}^n \times \mathbf{B}^m, \Upsilon^*(\mu,\nu) = \overline{\Upsilon(\overline{\mu},\overline{\nu})}$. *Here the bar* $\overline{\mu}$ *refers to the complement done coordinatewise.*

Definition 21. *The* ***dual*** *of the system* f *is by definition the system* $f^* : U^* \to P^*(S^{(n)})$, *where* $U^* = \{\overline{u} | u \in U\}$ *and* $\forall u \in U^*, f^*(u) = \{\overline{x} | x \in f(\overline{u})\}$.

Remark 22. *The system* f^* *models the circuit modeled by* f *with the AND gates replaced by OR gates etc.*

Notation 23. *We denote* $i_{f^*} : U^* \to P^*(\mathbf{B}^n)$, $\forall u \in U^*, i_{f^*}(u) = \{\overline{\mu} | \mu \in i_f(\overline{u})\}$.

Notation 24. *We denote by* $\pi_{f^*} : W_{f^*} \to P^*(P_n)$ *where* $W_{f^*} = \{(\overline{x(-\infty+0)}, u) | u \in U^*, x \in f(\overline{u})\}$ *the function* $\forall(\mu,u) \in W_{f^*}, \pi_{f^*}(\mu,u) = \pi_f(\overline{\mu},\overline{u})$.

Theorem 25. *The dual system* f^* *of* $f \subset \Sigma^-_{\Upsilon}$ *is regular,* $f^* \subset \Sigma^-_{\Upsilon^*}$; *its initial state function is* i_{f^*} *and its computation function is* π_{f^*}.

4 Cartesian product

Definition 26. *The* ***Cartesian product*** *of the systems* f *and* $f' : U' \to P^*(S^{(n')}), U' \in P^*(S^{(m')})$ *is defined as* $f \times f' : U \times U' \to P^*(S^{(n+n')})$, $\forall(u,u') \in U \times U', (f \times f')(u,u') = f(u) \times f'(u')$.

Remark 27. *The Cartesian product* $f \times f'$ *models two circuits that run independently on each other.*

Notation 28. *For* Υ *and* $\Upsilon' : \mathbf{B}^{n'} \times \mathbf{B}^{m'} \to \mathbf{B}^{n'}$, *we denote by* $\Upsilon \times \Upsilon' : \mathbf{B}^{n+n'} \times \mathbf{B}^{m+m'} \to \mathbf{B}^{n+n'}$ *the function* $\forall((\mu,\mu'),(\nu,\nu')) \in \mathbf{B}^{n+n'} \times \mathbf{B}^{m+m'}, (\Upsilon \times \Upsilon')((\mu,\mu'),(\nu,\nu')) = (\Upsilon(\mu,\nu), \Upsilon'(\mu',\nu'))$. *In this notation we identify* $(\mu,\mu') \in \mathbf{B}^n \times \mathbf{B}^{n'}$ *with* $(\mu_1,...,\mu_n,\mu'_1,...,\mu'_{n'}) \in \mathbf{B}^{n+n'}$ *etc.*

Notation 29. *If* $i_{f'} : U' \to P^*(\mathbf{B}^{n'})$ *is the initial state function of* f', *we use the notation* $i_{f\times f'} : U \times U' \to P^*(\mathbf{B}^{n+n'}), \forall(u,u') \in U \times U', i_{f\times f'}(u,u') = i_f(u) \times i_{f'}(u')$.

Notation 30. *The regular systems* f, f' *are given,* $f \subset \Sigma_{\Upsilon}$, $f' \subset \Sigma_{\Upsilon'}$ *as well as their computation functions:* $\pi_f : W_f \to P^*(P_n)$, $\pi_{f'} : W_{f'} \to P^*(P_{n'})$. *We denote by* $\pi_{f\times f'} : W_{f\times f'} \to P^*(P_{n+n'})$ *the function* $W_{f\times f'} = \{((x(-\infty+0), x'(-\infty+0)),(u,u')) | (u,u') \in U \times U', (x,x') \in f(u) \times f'(u')\}, \forall((\mu,\mu'),(u,u')) \in W_{f\times f'}$, $\pi_{f\times f'}((\mu,\mu'),(u,u')) = \pi_f(\mu,u) \times \pi_{f'}(\mu',u')$.

Remark 31. *The function* $\pi_{f\times f'}$ *is correctly defined since* $\forall\rho,\forall\rho',\rho \in P_n$ *and* $\rho' \in P_{n'} \Longrightarrow (\rho,\rho') \in P_{n+n'}$.

Theorem 32. *If* $f \subset \Sigma^-_{\Upsilon}, f' \subset \Sigma^-_{\Upsilon'}$, *then the system* $f \times f'$ *is regular,* $f \times f' \subset \Sigma^-_{\Upsilon\times\Upsilon'}$; *its initial state function is* $i_{f\times f'}$ *and its computation function is* $\pi_{f\times f'}$.

5 Parallel connection

Definition 33. *The* ***parallel connection*** *of* f *and* $f'_1 : U'_1 \to P^*(S^{(n')})$, $U'_1 \in P^*(S^{(m)})$ *is defined whenever* $U \cap U'_1 \neq \emptyset$ *by* $f||f'_1 : U \cap U'_1 \to P^*(S^{(n+n')})$, $\forall u \in U \cap U'_1, (f||f'_1)(u) = f(u) \times f'_1(u)$.

Remark 34. *The parallel connection* $f||f'_1$ *models two circuits that run under the same input, independently on each other.*

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Notation 35. *Let be* Υ *and* $\Upsilon'_1 : \mathbf{B}^{n'} \times \mathbf{B}^m \to \mathbf{B}^{n'}$, *for which we denote by* $\Upsilon || \Upsilon'_1 : \mathbf{B}^{n+n'} \times \mathbf{B}^m \to \mathbf{B}^{n+n'}$ *the function* $\forall((\mu,\mu'),\nu) \in \mathbf{B}^{n+n'} \times \mathbf{B}^m, (\Upsilon||\Upsilon'_1)((\mu,\mu'),\nu) = (\Upsilon(\mu,\nu), \Upsilon'_1(\mu',\nu))$.

Notation 36. *Let* $i_{f'_1} : U'_1 \to P^*(\mathbf{B}^{n'})$ *be the initial state function of* f'_1. *If* $U \cap U'_1 \neq \emptyset$, *we use the notation* $i_{f||f'_1} : U \cap U'_1 \to P^*(\mathbf{B}^{n+n'}), \forall u \in U \cap U'_1, i_{f||f'_1}(u) = i_f(u) \times i_{f'_1}(u)$.

Notation 37. *We suppose that the systems* f, f'_1 *are regular i.e.* $f \subset \Sigma^-_{\Upsilon}, f'_1 \subset \Sigma^-_{\Upsilon'_1}$ *and let* $\pi_f : W_f \to P^*(P_n), \pi_{f'_1} : W_{f'_1} \to P^*(P_{n'})$ *be their computation functions. If* $U \cap U'_1 \neq \emptyset$, *then we use the notation* $\pi_{f||f'_1} : W_{f||f'_1} \to P^*(P_{n+n'})$, $W_{f||f'_1} = \{((x(-\infty+0), x'(-\infty+0)), u) | u \in U \cap U'_1, x \in f(u), x' \in f'_1(u)\}$, $\forall((\mu,\mu'),u) \in W_{f||f'_1}, \pi_{f||f'_1}((\mu,\mu'),u) = \pi_f(\mu,u) \times \pi_{f'_1}(\mu',u)$.

Theorem 38. *If* $f \subset \Sigma^-_{\Upsilon}$, $f'_1 \subset \Sigma^-_{\Upsilon'_1}$ *and* $U \cap U'_1 \neq \emptyset$, *then* $f||f'_1 \subset \Sigma^-_{\Upsilon||\Upsilon'_1}$; *its initial state function is* $i_{f||f'_1}$ *and its computation function is* $\pi_{f||f'_1}$.

6 Serial connection

Remark 39. *Let be the systems* f *and* $h : X \to P^*(S^{(p)}), X \in P^*(S^{(n)})$. *When* $\bigcup_{u \in U} f(u) \subset X$, *the serial connection of* f *and* h *is defined by* $h \circ f : U \to P^*(S^{(p)}), \forall u \in U, (h \circ f)(u) = \bigcup_{x \in f(u)} h(x)$. *If* f *and* h *are regular, this definition means that in the systems of equations*

$$\left\{ \begin{array}{c} x(-\infty+0) = \mu \\ \forall i \in \{1,...,n\}, x_i(t) = \left\{ \begin{array}{c} \Upsilon_i(x(t-0), u(t-0)), if\ \rho_i(t) = 1 \\ x_i(t-0), otherwise \end{array} \right. \end{array} \right. , \tag{3}$$

$$\left\{ \begin{array}{c} y(-\infty+0) = \lambda \\ \forall j \in \{1,...,p\}, y_j(t) = \left\{ \begin{array}{c} \vartheta_j(y(t-0), x(t-0)), if\ \varpi_j(t) = 1 \\ y_j(t-0), otherwise \end{array} \right. \end{array} \right. \tag{4}$$

where $u \in S^{(m)}, x \in S^{(n)}, y \in S^{(p)}, \mu \in \mathbf{B}^n$, $\lambda \in \mathbf{B}^p$, $\rho \in P_n$, $\varpi \in P_p$, $\Upsilon : \mathbf{B}^n \times \mathbf{B}^m \to \mathbf{B}^n$, $\vartheta : \mathbf{B}^p \times \mathbf{B}^n \to \mathbf{B}^p$ *we eliminate* x. *Because this does not give any information of the regularity of* $h \circ f$, *we choose to work with a slightly different system from* $h \circ f$, *for which* x *is not eliminated.*

Notation 40. *If* f *and* h *fulfill* $\bigcup_{u \in U} f(u) \subset X$, *then we denote by* $h * f : U \to P^*(S^{(n+p)})$ *the system* $\forall u \in U, (h * f)(u) = \{(x,y) | x \in f(u), y \in h(x)\}$.

Notation 41. *The function* $\vartheta * \Upsilon : \mathbf{B}^{n+p} \times \mathbf{B}^m \to \mathbf{B}^{n+p}$ *is defined by* $\forall((\mu,\lambda),\nu) \in \mathbf{B}^{n+p} \times \mathbf{B}^m, (\vartheta * \Upsilon)((\mu,\lambda),\nu) = (\Upsilon(\mu,\nu), \vartheta(\lambda, \Upsilon(\mu,\nu)))$.

Remark 42. *The point is that, instead of eliminating* x *in (3), (4) as* $h \circ f$ *does, we can work with* $h * f$ *and with the equation*

$$\left\{ \begin{array}{c} z(-\infty+0) = (\mu,\lambda) \\ \forall k \in \{1,...,n+p\}, z_k(t) = \left\{ \begin{array}{c} (\vartheta * \Upsilon)_k(z(t-0), u(t-0)), if\ (\rho,\varpi)_k(t) = 1 \\ z_k(t-0), otherwise \end{array} \right. \end{array} \right.$$

where $z \in S^{(n+p)}$.

Notation 43. *For* $i_h : X \to P^*(\mathbf{B}^p)$ *the initial state function of* h, *we denote by* $i_{h*f} : U \to P^*(\mathbf{B}^{n+p})$ *the function* $\forall u \in U, i_{h*f}(u) = \{(\mu,\lambda) | \mu \in i_f(u), \lambda \in \bigcup_{x \in f(u), x(-\infty+0)=\mu} i_h(x)\}$.

Notation 44. *We suppose that* $\pi_h : W_h \rightarrow P^*(P_p)$ *is the computation function of* h, $W_h = \{(y(-\infty+0), x) | x \in X, y \in h(x)\}$. *We denote by* $\pi_{h*f} : W_{h*f} \rightarrow P^*(P_{n+p})$ *the function* $W_{h*f} = \{((x(-\infty+0), y(-\infty+0)), u) | u \in U, x \in f(u), y \in h(x)\}$, $\forall((\mu, \lambda), u) \in W_{h*f}$, $\pi_{h*f}((\mu, \lambda), u) = \{(\rho, \varpi) | \rho \in \pi_f(\mu, u), \varpi \in \bigcup_{x \in f(u), x(-\infty+0)=\mu} \pi_h(\lambda, x)\}$.

Theorem 45. *The systems* f *and* h *are given such that the inclusion* $\bigcup_{u \in U} f(u) \subset X$ *is true. If the regularity properties* $f \subset \Sigma^-_\Upsilon, h \subset \Sigma^-_\vartheta$ *hold, then* $h * f \subset \Sigma^-_{\vartheta * \Upsilon}$; *the initial state function of* $h * f$ *is* i_{h*f} *and its computation function is* π_{h*f}.

7 Intersection

Definition 46. *The* ***intersection*** *of* $f : U \rightarrow P^*(S^{(n)})$ *and* $g : V \rightarrow P^*(S^{(n)}), U, V \in P^*(S^{(m)})$ *is defined whenever* $\exists u \in U \cap V, f(u) \cap g(u) \neq \emptyset$ *by* $f \cap g : W \rightarrow P^*(S^{(n)})$, $W = \{u | u \in U \cap V, f(u) \cap g(u) \neq \emptyset\}$, $\forall u \in W, (f \cap g)(u) = f(u) \cap g(u)$.

Remark 47. *The intersection of two systems is a model that results by the simultaneous validity of two compatible models.*

Notation 48. *When* $W \neq \emptyset$, *we use the notation* $i_{f \cap g} : W \rightarrow P^*(\mathbf{B}^n), \forall u \in W, i_{f \cap g}(u) = i_f(u) \cap i_g(u)$.

Notation 49. *We consider the regular systems* f, g *for which the generator function* $\Upsilon : \mathbf{B}^n \times \mathbf{B}^m \rightarrow \mathbf{B}^n$ *is given such that* $f \subset \Sigma^-_\Upsilon, g \subset \Sigma^-_\Upsilon$. *Their computation functions are* $\pi_f : W_f \rightarrow P^*(P_n)$, $\pi_g : W_g \rightarrow P^*(P_n)$. *If the set* W *is non-empty, then we use the notation* $\pi_{f \cap g} : W_{f \cap g} \rightarrow P^*(P_n)$ *for the function that is defined by* $W_{f \cap g} = \{(x(-\infty+0), u) | u \in W, x \in f(u) \cap g(u)\}, \forall(\mu, u) \in W_{f \cap g}$, $\pi_{f \cap g}(\mu, u) = \{\rho | \rho \in \pi_f(\mu, u), \exists \rho' \in \pi_g(\mu, u), \Upsilon^{-\rho}(t, \mu, u) = \Upsilon^{-\rho'}(t, \mu, u)\}$.

Remark 50. *We remark the satisfaction of the following property of symmetry:* $W_{f \cap g} = W_{g \cap f}$ *and* $\forall(\mu, u) \in W_{f \cap g}, \forall \rho \in \pi_{f \cap g}(\mu, u), \exists \rho' \in \pi_{g \cap f}(\mu, u), \Upsilon^{-\rho}(t, \mu, u) = \Upsilon^{-\rho'}(t, \mu, u)$ *and* $\forall \rho' \in \pi_{g \cap f}(\mu, u), \exists \rho \in \pi_{f \cap g}(\mu, u), \Upsilon^{-\rho'}(t, \mu, u) = \Upsilon^{-\rho}(t, \mu, u)$.

Theorem 51. *If the regular systems* $f \subset \Sigma^-_\Upsilon, g \subset \Sigma^-_\Upsilon$ *fulfill* $W \neq \emptyset$, *then their intersection* $f \cap g : W \rightarrow P^*(S^{(n)})$ *is regular* $f \cap g \subset \Sigma^-_\Upsilon$; *its initial state function is* $i_{f \cap g}$ *and its computation function is* $\pi_{f \cap g}$.

8 Union

Definition 52. *The* ***union*** *of* f, g *is defined by* $f \cup g : U \cup V \rightarrow P^*(S^{(n)})$, $\forall u \in U \cup V, (f \cup g)(u) = \begin{cases} f(u), u \in U \setminus V, \\ g(u), u \in V \setminus U, \\ f(u) \cup g(u), u \in U \cap V \end{cases}$.

Remark 53. *The union of the systems represents the validity of one of two models. This is useful for example in testing, when* f *is the model of the 'good' circuit and* g *is the model of the 'bad' circuit.*

Notation 54. *We denote by* $i_{f \cup g} : U \cup V \rightarrow P^*(\mathbf{B}^n)$ *the function* $\forall u \in U \cup V, i_{f \cup g}(u) = \begin{cases} i_f(u), u \in U \setminus V, \\ i_g(u), u \in V \setminus U, \\ i_f(u) \cup i_g(u), u \in U \cap V \end{cases}$.

Lemma 55. *The sets* $W_f = \{(x(-\infty+0),u) | u \in U, x \in f(u)\}$, $W_g = \{(x(-\infty+0),u) | u \in V, x \in g(u)\}$, $W_{f\cup g} = \{(x(-\infty+0),u) | u \in U \cup V, x \in (f\cup g)(u)\}$ *fulfill* $W_{f\cup g} = W_f \cup W_g$.

Notation 56. *Let be the regular systems* f, g *and the function* Υ *such that* $f \subset \Sigma^-_\Upsilon, g \subset \Sigma^-_\Upsilon$ *are true. The computation functions of* f, g *are* π_f, π_g. *We denote by* $\pi_{f\cup g} : W_{f\cup g} \to P^*(P_n)$ *the function* $\forall(\mu,u) \in W_{f\cup g}$, $\pi_{f\cup g}(\mu,u) = \begin{cases} \pi_f(\mu,u), (\mu,u) \in W_f \setminus W_g, \\ \pi_g(\mu,u), (\mu,u) \in W_g \setminus W_f, \\ \pi_f(\mu,u) \cup \pi_g(\mu,u), (\mu,u) \in W_f \cap W_g \end{cases}$.

Theorem 57. *If the systems* f, g *are regular* $f \subset \Sigma^-_\Upsilon, g \subset \Sigma^-_\Upsilon$, *then the union* $f \cup g : U \cup V \to P^*(S^{(n)})$ *is regular,* $f \cup g \subset \Sigma^-_\Upsilon$; *its initial state function is* $i_{f\cup g}$ *and its computation function is* $\pi_{f\cup g}$.

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